Department of Mathematics & Philosophy of Engineering

Faculty of Engineering Technology

The Open University of Sri Lanka

Course: MPZ 3132-Engineering Mathematics IB **Assignment No.01 Academic Year – 2011/2012**

Instructions

- Answer all questions
- Write your address back of your answer scripts
- Use both sides of paper when you are doing assignment.
- Please send the answer scripts of your assignment **on or before the due date** to the following address.

Course Coordinator – MPZ 3132,

Dept. of Mathematics & Philosophy of Engineering,

Faculty of Engineering Technology,

The Open University of Sri Lanka,

Nawala,

Nugegoda.

You can collect model answers from virtual class ***(www.ou.ac.lk)***

User name - student0 Password – MPZ3132

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1. Define the Fourier series

1.1. The function f is defined such that $f(x) = \begin{cases} 0 & -\pi \leq x \leq 0 \\ sinx & 0 \leq x \leq \pi \end{cases}$ with period 2π.

1.1.1. Draw the graph of $f(x)$ on $[\,-4\pi, 5\pi\,]$.

1.1.2. Prove that $f(x) = \frac{1}{\pi} + \frac{1}{2} sinx - \frac{2}{\pi} \sum_{r=1}^{\propto} \frac{cos\,2rx}{4r^2-1}$.

1.1.3. Hence deduce that $\frac{\pi-2}{4} = \sum_{r=1}^{\propto} \frac{(-1)^{r-1}}{4r^2-1}$.

1.2. The function g is defined such that $g(x) = x$ where $x \in (-1,1)$ with period 2.

1.2.1. Draw the graph of $g(x)$ on $[\,-4, 4\,]$.

1.2.2. Find the Fourier series representation of g.

1.2.3. Hence find the Fourier series representation of $h(x) = x^2, x \in (-1,1)$ with period 2.

2. State the Dirichlet conditions

2.1. The function f is defined as $f(x) = x(x+1)$ where $x \in (-\pi, \pi)$ and $f(x) = f(x + 2k\pi)$ where $k \in Z$.

2.1.1. Draw the graph of the function f in the interval $[-5\pi, 5\pi]$.

2.1.2. Prove that $f(x) = \frac{\pi^2}{3} + \sum_{r=1}^{\propto} 2(-1)^r \left[\frac{2}{r^2} cosrx - \frac{1}{r} sinrx\right]$.

2.1.3. Deduce that $\frac{\pi^2}{6} = \sum_{r=1}^{\propto} \frac{1}{r^2}$.

2.2. Given that $g(x) = \begin{cases} \frac{1}{2} - x & -\frac{1}{2} < x < 0 \\ \frac{1}{2} + x & 0 < x < \frac{1}{2} \end{cases}$ and $g(x) = g(x+k)$ where $k \in Z$.

2.2.1. Show that the function g is an even function.

2.2.2. Draw the graph of g in the interval $[-2, 2]$.

2.2.3. Prove that $g(x) = \frac{3}{4} - \frac{2}{\pi^2} \sum_{r=1}^{\propto} \frac{cos\, 2(2r-1)\pi x}{(2r-1)^2}$.

2.2.4. Deduce that $\frac{\pi^2}{8} = \sum_{r=1}^{\propto} \frac{1}{(2r-1)^2}$.

3. The function f is defined as $f(x) = \pi - x \quad x \in (0, \pi)$

3.1. Extend the above function as a

3.1.1. Odd periodic function with period 2π .

3.1.2. Even periodic function with period 2π.

3.1.3. Periodic function with period π.

3.2. Draw the graphs of the above three functions on $[-4\pi, 4\pi]$.

3.3. Prove that the Fourier representation of $f(x) = \pi - x \quad x \in (0, \pi)$

3.3.1. as a sine series with period 2π is $2\sum_{r=1}^{\propto} \frac{sinrx}{r}$.

3.3.2. as a cosine series with period 2π is $\frac{\pi}{2} + \frac{4}{\pi} \sum_{r=1}^{\propto} \frac{\cos(2r-1)x}{(2r-1)^2}$.

3.3.3. as a full trigonometric series with period π is $\frac{\pi}{2} + \sum_{r=1}^{\propto} \frac{\sin 2rx}{r}$.

3.3.4. Using Perceval identity deduce that $\frac{\pi^4}{96} = \sum_{r=1}^{\propto} \frac{1}{(2r-1)^4}$ and $\frac{\pi^2}{6} = \sum_{r=1}^{\propto} \frac{1}{r^2}$.

4. Define the Taylor polynomial of degree n about a

4.1. If $f(x) = \ln(1 - 2xcos\theta + x^2)$

4.1.1. Prove that$(1 - 2xcos\theta + x^2)\frac{d^5 f(x)}{dx^5} + 8(x - cos\theta)\frac{d^4 f(x)}{dx^4} + 12\frac{d^3 f(x)}{dx^3} = 0.$

4.1.2. Prove that the Taylor polynomial of order five of $f(x)$ is – $2\sum_{r=1}^{5}\frac{cosr\theta\, x^r}{r}$.

4.2. If $f(x) = \ln(cosx)$ prove that $\frac{d^3 f(x)}{dx^3} + 2\frac{d^2 f(x)}{dx^2}\frac{df(x)}{dx} = 0.$

4.2.1. Hence find Taylor polynomial of degree four of $f(x) = \ln(cosx)$ about $x = 0.$

4.2.2. Deduce that $ln2 \approx \frac{\pi^2}{16}\left(1 + \frac{\pi^2}{96}\right)$.

5. Define the Taylor series expansion about a

5.1. Prove that the Taylor series expansion about $x = 0$ of $sinx$ and $cosx$ are $\sum_{r=0}^{\propto}\frac{(-1)^r x^{2r+1}}{(2r+1)!}$ and $\sum_{r=0}^{\propto}\frac{(-1)^r x^{2r}}{(2r)!}$ respectively.

5.1.1. Find the Taylor series expansions of sin^2x and cos^2x about $x = 0.$

5.1.2. Deduce that $\int_0^x sin(x^2)dx = \sum_{r=0}^{\propto}\frac{(-1)^r x^{4r+3}}{(4n+3)(2r+1)!}$

5.2. Given that $g(x) = \frac{1}{x}$

5.2.1. Prove that Taylor series expansion of $\frac{1}{x}$ about $x = 3$ is $\sum_{r=0}^{\infty}\frac{(-1)^r (x-3)^r}{3^{r+1}}$.

5.2.2. Show also that the above series converges to $\frac{1}{x}$ and find the radius of convergence. (Hint: Use the convergence of a geometric series.)

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